Jan. 2022
Maths 1

Roll No. [ ] Total No. of Pages : 02

Total No. of Questions : 09

B.Tech. (CE/ME/ECE/EE) (2018 Batch) (Sem.–1)

MATHEMATICS-I

Subject Code : BTAM-101-18

M.Code : 75353

Time : 3 Hrs. Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions from SECTION - B & C.

SECTION-A

Q1 a) Give geometric interpretation of Rolle's theorem.

b) Can Rolle's theorem be applied to the function $f(x) = \tan x$ in the interval $[0, \pi]$.

c) Evaluate $\lim\limits_{x \to 0} \dfrac{\ln(1+x)}{\sin x}$.

d) Does the limit $\lim\limits_{(x,y)\to(0,0)} \dfrac{x+\sqrt{y}}{x^2+y^2}$ exists?

e) Give the coordinates of center of mass of solid of mass M.

f) Define convergence, divergence and oscillation of a sequence.

g) Define Cauchy's root test to check the convergence of the positive term series $\sum u_n$.

h) Find sum and product of Eigen values of the matrix $\begin{bmatrix} -2 & 1 \\ -3 & 1 \end{bmatrix}$.

i) Find the inverse of the matrix $\begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$.

(j) Find rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$.

SECTION-B

Q2 a) Expand $f(x) = \log \sin x$ in powers of $(x-3)$.

b) Evaluate the limit $\lim_{x \to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$

Q3. a) Find the volume of the solid generated by the revolution of the curve $y(a^2 + x^2) = a^3$ about its asymptote.

b) Find extremum of the function $\sin x + \cos x$.

Q4. a) Discuss the continuity of the function $f(x, y) = \begin{cases} \frac{(x-y)^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ at $(0, 0)$.

b) Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $xyz = a^3$.

Q5. a) Evaluate the integral $\iint_R xy\,dxdy$, where R is the region bounded by the x axis, the line $y = 2x$ and the parabola $4ay = x^2$.

b) Evaluate the integral $\iiint_T x^2 y^2 z\,dxdydz$, over the boundary of $T: x^2 + y^2 \leq 1, 0 \leq z \leq 1$.

SECTION-C

Q6. Show that the GP, $1 + r + r^2 + r^3 + \ldots \infty$ (i) converges if $|r| < 1$, (ii) diverges if $r \geq 1$ and (iii) oscillates if $r < -1$.

Q7. a) Examine the convergence of the series $\sum \frac{2n^3+5}{4n^5+1}$.

b) Examine the convergence of the alternating series $\frac{1}{1.2} - \frac{1}{3.4} + \frac{1}{5.6} - \frac{1}{7.8} + \ldots$

Q8. Find the characteristic equation of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence compute A^{-1}. Find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$.

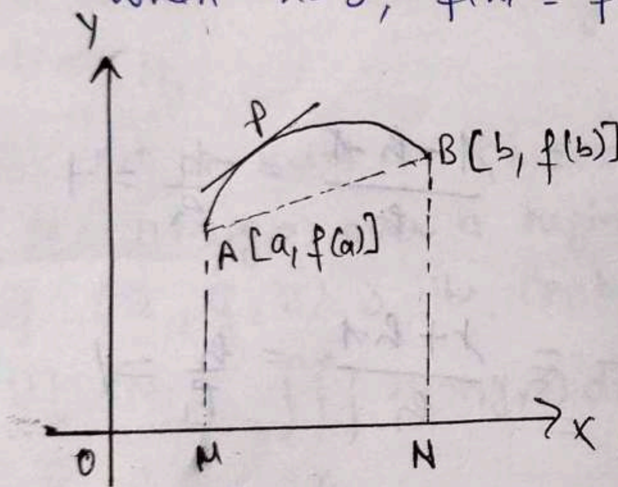
Q9. Reduce the matrix $\begin{bmatrix} 4 & 2 & 1 \\ 6 & 3 & 4 \\ 2 & 1 & 0 \end{bmatrix}$ to the diagonal form.

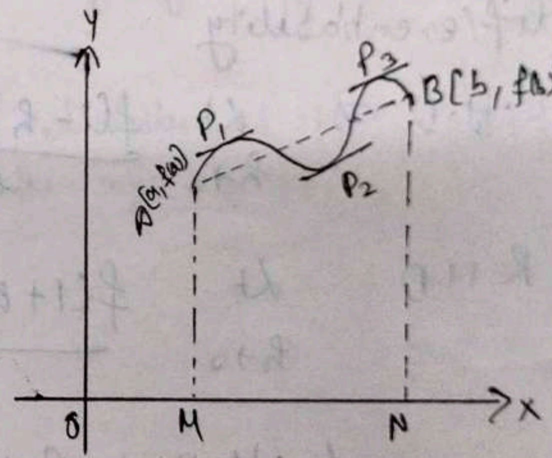
NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.

B. Tech (Sem - 1)
Mathematics - I
Subject Code : BTAM-101-18
M.code : 7
Date of Examination: 11-01-2023

Q1 (a) Give geometric interpretation of Mean Value Theorem.

Sol: When $x=a$, $f(x) = f(a)$
When $x=b$, $f(x) = f(b)$





$\therefore$ $(a, f(a))$, $(b, f(b))$ are two points on the graph of $y = f(x)$.
Let these points be A & B.
$\because$ $f(x)$ is continuous in $a \le x \le b$. $\therefore$ graph of $f(x)$ is continuous for $a \le x \le b$
$\because$ $f(x)$ is derivable for $a < x < b$.
Therefore tangent exists at each point of the graph for $a < x < b$
$\therefore$ there exists atleast one point P on graph between the point A & B such that the tangent at P is parallel to the chord AB. If C is abscissa of P, then

$$\boxed{f'(c) = \text{Slope of Chord } AB = \frac{f(b) - f(a)}{b - a}}$$

(b) Can Rolle's theorem be applicable to the function

$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2-x, & 1 \le x \le 2 \end{cases} \quad \text{in } [0, 2]$$

Sol: We will firstly check whether the function is diff. or not as well as continuous or not.

$$L.H.L. = \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x = 1$$

$$R.H.L = \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 2-x = 2-1 = 1$$

Also $\lim_{x \to 1} f(x) = 1$

∴ the function is continuous, Now we will check differentiability

$$L.H.D \text{ is } \lim_{h \to 0} \frac{f(1-h) - f(1)}{h} = \frac{\cancel{1} - h - \cancel{1}}{h} = \frac{-h}{h} = -1$$

$$R.H.D \quad \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \frac{\cancel{1} + h - \cancel{1}}{h} = \frac{h}{h} = 1$$

$$L.H.D \ne R.H.D.$$

∴ the function is not differentiable

Hence Rolle's theorem is not applicable.

(2)

Does the limit $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ exists?

Let us take the general path $y = mx$ to simplyfy function

$$\underset{x\to 0}{\text{Limit}} \frac{x(mx)}{x^2+(mx)^2} = \underset{x\to 0}{Lt} \frac{mx^2}{x^2+m^2x^2}$$

$$= \underset{x\to 0}{Lt} \frac{x^2 m}{x^2(1+m^2)}$$

$$= \frac{m}{1+m^2}.$$

$\therefore$ The limit does not exist.

Give the coordinates of the Center of gravity of Solid of Mass M.

Let a mass M be continuously distributed with density $\mu = \mu(x,y,z)$ over a region A of the xyz-space.

If $(\bar{x}, \bar{y}, \bar{z})$ is the centroid, then

(1) $M = \iiint_A \mu(x,y,z)\, dx\,dy\,dz.$

(2) $\bar{x} = \frac{1}{M} \iiint_A x\mu(x,y,z)\, dx\,dy\,dz$

(3) $\bar{y} = \frac{1}{M} \iiint_A y\mu(x,y,z)\, dx\,dy\,dz$

(4) $\bar{z} = \frac{1}{M} \iiint_A z\mu(x,y,z)\, dx\,dy\,dz.$

f) Define Convergence, divergence and Oscillation of Series.

Sol Convergent Series: The series Σa_n is said to converge if the sequence $\{S_n\}$ of its partial sums converges to s and s is called the sum of the convergent infinite series $\sum_{n=1}^{\infty} a_n$

We write $\sum_{n=1}^{\infty} a_n = s$.

(ii) Divergent Series : The series Σa_n is said to diverge to ∞ if the sequence $\{S_n\}$ diverges to ∞. We write $\sum_{n=1}^{\infty} a_n = \infty$.

(iii) Oscillatory Series: The series Σa_n is said to oscillate finitely or infinitely if the sequence $\{S_n\}$ oscillates finitely or infinitely. In other words, if the series Σa_n neither converges nor diverges to $+\infty$ or $-\infty$, it is said to oscillates finitely or infinitely.

$f(x) = \frac{x}{e}$

By using

(g) D' Alembert's Ratio Test : If $\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = l$, then the positive terms series Σa_n converges if $l<1$ and diverges if $l>1$.

(h) Find the sum and product of eigen values of the matrix $\begin{bmatrix} 1 & -1 \\ 2 & -5 \end{bmatrix}$

Sol Let $A = \begin{bmatrix} 1 & -1 \\ 2 & -5 \end{bmatrix}$

Sum of eigen values = Sum of its diagonal elements.

$= 1-5 = -4$

Product of eigen values $= \begin{bmatrix} 1 & -1 \\ 2 & -5 \end{bmatrix} = -5+2 = -3.$

(i) Find the inverse of the matrix $\begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$.

Sol let $A = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$

$|A| = 12-1 = 11 \neq 0$ A^{-1} exist.

$A_{11} = (-1)^{1+1}4 = 4$ $A_{12} = (-1)^3 1 = -1$ $A_{21} = (-1)^3 \times 1 = -1$ $A_{22} = 3(-1)^4 = 3$

$\begin{bmatrix} 4 & -1 \\ -1 & 3 \end{bmatrix}$ $\therefore$ adj $A = \begin{bmatrix} 4 & -1 \\ -1 & 3 \end{bmatrix}$

$A^{-1} = \frac{adj A}{|A|} = \frac{1}{11}\begin{bmatrix} 4 & -1 \\ -1 & 3 \end{bmatrix}$ Ans

SECTION – B

③

$f(x) = e^x$ in Power of $(x-1)$ upto four term.

By USING Taylor's theorem for the function $f(x)$ ascending Power of $(x-a)$ is

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!} f''(a) + f'''(a)\frac{(x-a)^3}{3!}$$

Here $f(x) = e^x$ and $a = 1$

$f'(x) = e^x \qquad f'(1) = e^1 = e$

$f''(x) = e^x \qquad f''(1) = e^1 = e$

$f'''(x) = e^x \qquad f'''(1) = e^1 = e$

$$\therefore e^x = e + (x-1)e + \frac{(x-1)^2}{2!} e + \left(\frac{x-1}{3!}\right)^3 e$$

$$= e\left\{1 + (x-1)^1 + \frac{(x-1)^2}{2!} + \frac{(x-1)^2}{3!}\right\}$$

b.) Evaluate the limit $\lim_{x \to 0} \frac{e^x - e^{-x} - 2\log(1+x)}{x \sin x}$

$$\lim_{x \to 0} \frac{e^0 - e^0 - 2\log 1}{0} = \frac{0}{0} \text{ form}$$

$$\rightarrow \frac{e^x - e^{-x} - 2\log(1+x)}{x}\left[\because \frac{x}{\sin x}\right]$$

$\lim_{x\to 0} \frac{e^x - e^{-x} - 2\log(1+x)}{x^2}$

(38)

④

USING L-HOSPITAL RULE

$$\lim_{x\to 0} \frac{e^x + e^{-x} - 2/_{1+x}}{2x} = \frac{e^0 + e^0 - 2}{0} = 0/_0 \text{ form}$$

BY Again USING L-HOSPITAL

$$\lim_{x\to 0} \frac{e^x - e^{-x} + 2/(1+x)^2}{2} = \frac{e^0 - e^0 + 2}{2} = \underline{\underline{1}}$$

3.)

$$V = \pi \int_a^b y^2 \, dx$$

$$y^2(a+x) = x^2(3a-x)$$

$$y = 0, x = 0 \text{ and } x = 3a$$

$$V = \pi \int_0^{3a} \frac{x^2(3a-x)}{a+x} \, dx$$

Put $a+x = t$

$x = 0, t = a$

$x = 3a, t = 4a$

$$V = \pi \int_a^{4a} \frac{(t-a)^2 (3a-t+a)}{t} \, dt$$

$$\pi \int_a^{4a} \frac{(t^2 + a^2 - 2at)(3a - t + a)}{t} \, dt$$

Evaluate the limit $\underset{x\to 0}{Lt}\ \dfrac{e^x - e^{-x} - 2\log(1+x)}{x\sin x}$

Sol $\underset{x\to 0}{Lt}\ \dfrac{e^x - e^{-x} - 2\log(1+x)}{x\sin x} = \underset{x\to 0}{Lt}\ \dfrac{e^x - e^{-x} - 2\log(1+x)}{x^2}\cdot\dfrac{x}{\sin x}$

$= \underset{x\to 0}{Lt}\ \dfrac{e^x - e^{-x} - 2\log(1+x)}{x^2}\cdot 1$ $\left(\frac{0}{0}\text{ form}\right) \because \underset{x\to 0}{Lt}\ \dfrac{x}{\sin x} = 1$

Apply L'H Rule

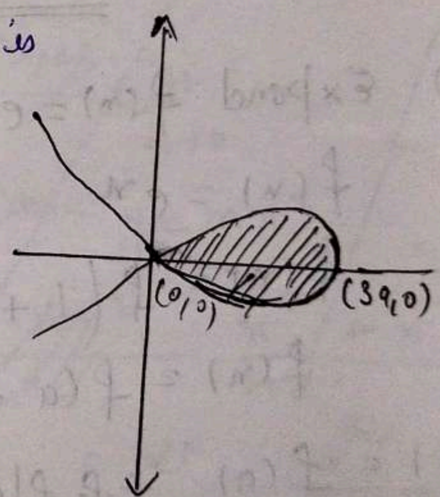
$= \underset{x\to 0}{Lt}\ \dfrac{e^x + e^{-x} - \frac{2}{1+x}}{2x}$ $\left(\frac{0}{0}\text{ from}\right)$

Again ~~Apply~~ Apply L.H. Rule $\Rightarrow \underset{x\to 0}{Lt}\ \dfrac{e^x - e^{-x} + \frac{2}{(1+x)^2}}{2}$

$= \dfrac{1-1+\frac{2}{1}}{2} = \dfrac{2}{2} = 1$ Ans

Q 3 (a) find the volume of the loop generated by the revolving the curve $y^2(a+x) = x^2(3a-x)$ about the x-axis.

Sol: The volume ~~generated~~ formed by revolution of loop about x-axis is



$= \int \pi y^2\, dx$

$= \pi \int_0^{3a} \dfrac{x^2(3a-x)}{(a+x)}\, dx$

$x^2(3a-x)$ can be expressed as $4a^3 - (x-2a)^2(a+x)$.

$\Rightarrow \pi \int_0^{3a} \left[-x^2 + 4ax - 4a^2 + \dfrac{4a^3}{a+x}\right] dx$

$= \pi \left[\dfrac{-x^3}{3}\right]_0^{3a} + 4a\left[\dfrac{x^2}{2}\right]_0^{2a} + 4a^2\left[x\right]_0^{3a} + 4a^3\left[\log(a+x)\right]_0^{3a}$

$= \pi\left[-3a^3 + 4a^3(\log 4a - \log a)\right]$

$= \pi a^3\left[4\log\dfrac{4a}{a} - 3\right] = \pi a^3\left(4\log(2)^2 - 3\right)$

$= \pi a^3(8\log 2 - 3)$ Ans//

(j) find rank of the matrix $\begin{bmatrix} 0 & 1 & -3 \\ 1 & 0 & 1 \\ 3 & 1 & 0 \end{bmatrix}$

Sol Let $A = \begin{bmatrix} 0 & 1 & -3 \\ 1 & 0 & 1 \\ 3 & 1 & 0 \end{bmatrix}$ ~~Now~~

Now, we will apply the row transformation

$$\sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -3 \\ 3 & 1 & 0 \end{bmatrix} \quad R_2 \leftrightarrow R_1$$

$$\sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -3 \\ 0 & 1 & -3 \end{bmatrix} \quad R_3 \to R_3 - 3R_1$$

$$\sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

$\therefore \rho(A) = 2.$

Section - B

Q2 (a) Expand $f(x) = e^x$ in powers of $(x-1)$ upto four terms.

Sol: $f(x) = e^x$

$\therefore f(x) = f(1 + (x-1))$

$f(x) = f(a + h) \Rightarrow a = 1,\ h = (x-1)$

$$f(x) = f(a) + h f'(a) + \frac{h^2}{2!} f''(a) + \frac{h^3}{3!} f'''(a) + \frac{h^4}{4!} f^{iv}(a) \ldots$$

$$f(x) = e^x = f(1) + h f'(1) + \frac{h^2}{2!} f''(1) + \frac{h^3}{3!} f'''(1) + \frac{h^4}{4!} f^{iv}(1)$$

$f(1) = e^1 = e$

$f'(x) = e^x,\ f'(1) = e^1$

$f''(x) = e^x,\ f''(1) = e$

$f'''(x) = e^x,\ f'''(1) = e^1$

$f^{iv}(1) = e^x,\ f^{iv}(1) = e^1$

$\therefore$

$$e^x = e + (x-1)e + \frac{(x-1)^2}{2!}e + \frac{(x-1)^3}{3!}e + \frac{(x-1)^4}{4!}e + \ldots \quad \text{Ans}$$

Find extremum of the function $2\sin x + \cos 2x$, $0 \le x \le 2\pi$. (4)

: $f(x) = 2\sin x + \cos 2x$, $0 \le x \le 2\pi$

$f'(x) = 2\cos x - 2\sin 2x$

$f'(x) = 0$

$2\cos x\,(1 - 2\sin x) = 0$

$x = \frac{\pi}{6}, \frac{\pi}{2}$

Now $f''(x) = -2\sin x - 4\cos 2x$

$f''\left(\frac{\pi}{6}\right) = -2\cdot\frac{1}{2} - 4\cdot\frac{1}{2} = -3 < 0$

$f''\left(\frac{\pi}{2}\right) = -2 - 4(-1) = 2 > 0$

Hence, $f(x)$ is maximum at $x = \pi/6$

Q4 (a) Discuss the continuity of the function $f(x,y) = \begin{cases} \frac{x^2+y^2}{xy}, & (x,y) \ne (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ at $(0,0)$

Sol: $f_x(0,0) = \lim_{h\to 0} \frac{f(0+h,0) - f(0,0)}{h} = \lim_{h \to 0}$

Let $(x,y) \to (0,0)$ along the curve $y = mx^2$.

$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \cancel{\frac{x^2\cdot mx^2}{x^4}}\ \frac{x^2 + m^2x^4}{mx^3}$

$= \lim_{(x,y)\to(0,0)} \frac{x^2(1+m^2x^2)}{mx^3}$

$\frac{1}{0} = \infty$

(7)

$\therefore f''(x) = -2\sin x - 4\cos 2x$

$\cancel{f''(\frac{\pi}{6}) = -2\times\frac{1}{2} - 4\times\frac{1}{2} = -3 < 0}$

$\cancel{f''(\pi/2) = -2 - 4(-1) = 2 > 0}$

Hence, $f(x)$ is Maximum at $\underline{x = \pi/6}$

4(a) :- The Continuity of the function $f(x,y) = \begin{cases} \frac{x^2+y^2}{xy}, & (x,y) \neq (0,0) \\ 0, & (x,y) \neq (0,0) \end{cases}$

$$\lim_{x\to 0}\left(\lim_{y\to 0} \frac{x^2+0}{0}\right) = \lim_{x\to 0}\left(\frac{x^2}{0}\right) = \frac{0}{0}$$

$$\lim_{y\to 0}\left(\lim_{x\to 0}\left(\frac{0+y^2}{0}\right)\right) = \lim_{x\to 0}\left(\frac{y^2}{0}\right) = \frac{0}{0}$$

L.H.S = R.H.S

$\rightarrow y = mx$

$$\lim_{x\to 0} \frac{x^2+m^2x^2}{x\cdot mx} = \frac{x^2(1+m^2)}{x^2} = 1+m^2 \neq 0$$

It is not A UNIQUE VALUE

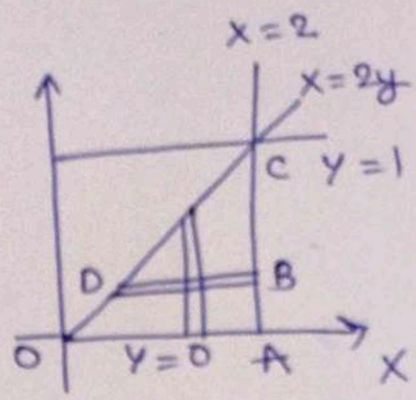
That's WHY this function is not Continuous.

b)

Evaluate the integral $\int_0^1 \int_{2y}^{2} e^{x^2}\,dx\,dy$ (8)

$R = \{(x,y) ; 0 \le y \le 1 , 2y \le x \le 2$

$x = 2y , x = 2 , y = 0 , y = 1$



Region R can be written as R =

$\{(x,y) ; 0 \le x \le 2 , 0 \le y \le \frac{x}{2}\}$

$$I = \int_0^2 \int_0^{x/2} e^{x^2}\,dy\,dx = \int_0^2 \left[e^{x^2} \cdot \frac{x}{2}\right] dx = \frac{1}{4}\int_0^3 \left[e^{x^2} \cdot 2x\,dx\right]$$

$$= \frac{1}{4}\left[e^{x^2}\right]_0^2 = \frac{1}{4}\left[e^4 - 1\right]$$

b.) $\int_0^1 \int_0^{x^2} \int_0^{(x+y)} (x+3y-2z)\,dz\,dy\,dx$

C

$$= \cancel{\int_0^1 \int_0^{x^2} \left[\frac{-2z^2}{2}\right]_0^{x+y}}$$

$$\int_0^1 \int_0^{x^2} (xz + 3yz - z^2)_0^{x+y}$$

$$\int_0^1 \int_0^{x^2} x(x+y) + 3y(x+y) - (x+y)^2$$

$$= \int_0^1 \int_0^{x^2} \cancel{x^2} + xy + 3yx + 3y^2 - \cancel{x^2} - y^2 - 2xy$$

$$= \int_0^1 (2y^2 + 2yx)_0^{x^2}$$

$$\sum \frac{3\cdot 6\cdot 9 \cdots 3n}{7\cdot 10\cdot 13 \cdots (3n+4)}\, x^n$$

$$a_n = \frac{3n}{3n+4}\, x^n$$

$$a_{n+1} = \frac{3n+1}{3n+7}\cdot x^{n+1}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{3n}{3n+4}\cdot\frac{3n+7}{3n+1}\cdot\frac{\cancel{x^n}\cdot x}{\cancel{x^n}}$$

$$\lim_{n\to\infty} \frac{9n^2+21n}{9n^2+3n+12n+4}\cdot x$$

$$= \frac{9+\frac{21}{n}}{9+\frac{3}{n}+\frac{12}{n}+\frac{4}{n^2}}\cdot x$$

$$\to \frac{9}{9}\cdot x = \underline{\underline{x}}$$

Now:-

$$\frac{a_n}{a_{n+1}} - 1 = \frac{3n}{3n+4}\cdot\frac{3n+7}{3n+1} - 1$$

$$= \frac{9n^2+21n}{9n^2+3n+12n+4} - 1$$

(10)

$$\lim_{n\to\infty} \frac{\cancel{9n^2}+21n-\cancel{9n^2}-15n-4}{9n^2+15n+4} x \lim_{n\to\infty} n\left(\frac{6n-4}{9n^2+15n+4}\right)$$

$$\to \lim_{n\to\infty} \frac{6-\frac{4}{n}}{9+\frac{15}{n}+\frac{4}{n^2}} \cdot x = \frac{\cancel{6}}{\cancel{9}}\frac{2}{3}x<1$$

By Rabbie's Test Σa_n diverges

$$= \int_0^1 2(x^2)^2 + 2x^2 \cdot x$$

$$= \int_0^1 2x^4 + 2x^3 = 2\int_0^1 \frac{x^5}{5} + \frac{2x^4}{4}$$

$$= 2\left[\frac{1}{5} + \frac{2}{4}\right]$$

$$= 2\left[\frac{4+10}{20}\right] = \frac{14}{10}\ \underline{\underline{7/5}}$$

SECTION - C

7(a) $\frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + \frac{1}{3\cdot4\cdot5} + \cdots$

$$\Sigma a_n = \frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + \frac{1}{3\cdot4\cdot5} + \cdots$$

where $a_n = \dfrac{1+(n-1)2}{[1+(n-1)\cdot1][2+(n-1)\cdot1][3+(n-1)\cdot1]}$

$$a_n = \frac{2n-1}{n(n+1)(n+2)}$$

Let $b_n = \dfrac{1}{n^2}$

$$\frac{a_n}{b_n} = \frac{2n-1}{n(n+1)(n+2)} \times \frac{n^2}{1} = \frac{n(2n-1)}{(n+1)(n+2)} = \frac{2-\frac{1}{n}}{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)}$$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{2-0}{(1+0)(1+0)} = \underline{2}$ which is non zero & finite (12)

By Comparison test, Σa_n & b_n Converges or diverge together

But $\Sigma b_n = \Sigma \frac{1}{n^2}$ is Convergent

$\therefore$ Given series Σa_n is ~~Constant~~ (convergent)

b) $1 - \frac{1}{2^k} + \frac{1}{3^k} - \frac{1}{4^k} + \cdots\cdots$, for $k > 0$

$$\Sigma \frac{(-1)^{n-1}}{n^k} = 1 - \frac{1}{2^k} + \frac{1}{3^k} - \frac{1}{4^k} +$$

Comparing $\Sigma \frac{(-1)^{n-1}}{n^k}$ with $\Sigma (-1)^{n-1}$, we get $a_n = \frac{1}{n^k}$

i) $a_n > 0 \ \forall n$

ii) $a_n > a_{n+1} \ \forall n$

iii) $a_n = \frac{1}{n^k} \to 0$ as $n \to \infty$

BY LEIBNITZ TEST (1) is convergent.

(13)

$\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

$A - \lambda I = 0$

$= \begin{bmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{bmatrix}$

$\rightarrow (1-\lambda)(3-\lambda) - 8 = 0$

$= 3 - \lambda - 3\lambda + \lambda^2 - 8 = 0$

$\lambda^2 - 4\lambda - 5 = 0$

$A^2 - 4A - 5I = 0$

Multiply Both side by A^{-1}

$A - 4I - 5IA^{-1} = 0$

$-5IA^{-1} = -A + 4I$

$5A^{-1} = A - 4I$

$A^{-1} = \frac{1}{5}\left[\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}\right]$

$A^{-1} = \frac{1}{5}\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}$

$\rightarrow A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$

$A^3[A^2 - 4A - 7\cancel{A}] + 11A^2 - A - 10I$

$A^3[\underbrace{A^2 - 4A - 5I}_{0} - 2I] + 11A^2 - A - 10I$

$-2A^3 + 11A^2 - A - 10I$

$\rightarrow$

$2\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$

$\rightarrow -2\begin{bmatrix} 41 & 84 \\ 42 & 83 \end{bmatrix} + 11\begin{bmatrix} 9 & 16 \\ 8 & 17 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$

$= \begin{bmatrix} 82 & 168 \\ 84 & 166 \end{bmatrix} + \begin{bmatrix} 99 & 176 \\ 88 & 187 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ 2 & 8 \end{bmatrix}$

$$\begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix}$$

$$|A| = 5(260-4) - 3(30-14) + 7(6-182)$$

$$= 1280 - 1280 = \underline{\underline{0}}$$

$$\lambda^3 - (\text{Traces of } A)\lambda^2 + \left(\begin{matrix}\text{SUM of Minors of} \\ \text{diagonal}\end{matrix}\right)\lambda - |A| = 0$$

$$\lambda^3 - 41\lambda^2 + (256 + 1 + 121)\lambda - 0$$

$$\lambda^3 - 41\lambda^2 + 378\lambda = 0$$